THE COMPLEX MULTICHANNEL LMS ALGORITHM FOR ADAPTIVE BLIND SYSTEM IDENTIFICATION

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ABSTRACT

Existing adaptive algorithms for blind SIMO system identification are implicitly derived for real signals. In this paper, we extend the multichannel LMS algorithm to the complex case. This is useful, for example, in multirate implementations of the algorithms where the subband signals are usually complex. With this algorithm, the channels are identified correctly up to a complex multiplicative factor resulting in magnitude and phase ambiguity. The complications arising due to this phase ambiguity are discussed and simulation results are presented to demonstrate and to validate the performance of the complex multichannel LMS algorithm.

1. INTRODUCTION

A class of adaptive algorithms for blind identification of SIMO systems has been proposed with implementations both in the time domain [1] and in the frequency domain [2]. In the derivation of these algorithms, it is implicitly assumed that both the signals and the unknown systems are real valued. However, in some cases there is a need for complex adaptive filters. Such cases include blind system identification in subbands where the subband signals may be complex [3] and in particular for oversampled filter banks with an arbitrary oversampling factor [4]. There are also other applications of interest in, for example, communications [5, 6].

The advantages of oversampled subbands have been demonstrated for acoustic echo cancellation and the ability to implement blind adaptive system identification algorithms in subbands would be of great interest for speech dereverberation as was shown by Gannot and Moonen [7] for a subspace solution with real subband signals. However, to achieve this goal there are several issues to consider: i) implementation of complex adaptive blind channel identification algorithms, ii) the problem of determining the filter order in the subband equivalent filters and iii) the gain ambiguity inherent in blind system identification.

In this paper, we address the first of these issues. We adopt the approach by Widrow et al in [8] for the derivation of the complex LMS adaptive algorithm and extend the work by Huang and Benesty in [1] to derive the complex Multichannel LMS (MCLMS) algorithm for blind adaptive identification of SIMO systems. As in the case of real signals and systems, the unknown impulse responses are identified up to an arbitrary constant factor. However, this factor is now a complex number, which introduces an arbitrary magnitude and phase ambiguity in the identified channel coefficients. The implications of the phase ambiguity are discussed.

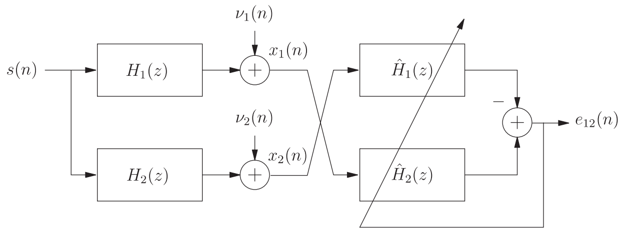

Figure 1: *System diagram for multichannel adaptive blind system identification based on the cross-relation between channels for $M = 2$ sensors.*

The remainder of this paper is organised as follows. The general problem of blind SIMO system identification is formulated in Section 2. The Complex MCLMS is derived in Section 3 followed by a discussion in Section 4 regarding the issues arising from the arbitrary complex factor. In Section 5 simulation results demonstrate the performance of the algorithm and finally, in Section 6, conclusions are drawn.

2. PROBLEM FORMULATION

In a SIMO system, a signal $s(n)$ is observed in a noisy multipath environment by an array of sensors at a distance from the source. The signal received at the lth sensor is

$$x_l(n) = \mathbf{h}_l^T \mathbf{s}(n) + \nu_l(n), \tag{1}$$

where $\mathbf{h}_l = [h_{l,0}\ h_{l,1}\ \ldots\ h_{l,L-1}]^T$ is the L-tap impulse response of the channel between the source and the lth sensor, $\mathbf{s}(n) = [s(n)\ s(n-1)\ \ldots\ s(n-L+1)]^T$ is the source signal vector and $\nu_l(n)$ is measurement noise at the

lth sensor. It is assumed that the signals and the channel coefficients are complex variables.

The aim of a blind channel identification algorithm is to form an estimate $\hat{\mathbf{h}}_l = [\hat{h}_{l,0}\ \hat{h}_{l,1}\ \ldots\ \hat{h}_{l,L-1}]^T$ of the impulse responses $\mathbf{h}_l$, using only the observations $x_l(n),\ l = 1, 2, \ldots, M$. This is possible provided that the following identifiability conditions are satisfied [9]: i) the channels do not share any common zeros and ii) the autocorrelation matrix of the source signal is of full rank.

3. THE COMPLEX MULTICHANNEL LMS

The multichannel LMS algorithm proposed in [1] is based on the cross-relation between two channels [1, 9] $x_1 * h_2 = s * h_1 * h_2 = x_2 * h_1$, which, in the noise-free case, leads to the relation

$$\mathbf{x}_l^T(n)\mathbf{h}_m = \mathbf{x}_m^T(n)\mathbf{h}_l, \quad l, m = 1, 2, \ldots, M \tag{2}$$

where $\mathbf{x}_l(n) = [x_l(n)\ x_l(n-1)\ \ldots\ x_l(n-L+1)]^T$ is a vector of observation samples at the lth sensor at time n. As in the case of real signals [1], in the presence of noise a complex error function can be defined

$$\begin{aligned} e_{lm}(n) &= \mathbf{x}_l^T(n)\hat{\mathbf{h}}_m - \mathbf{x}_m^T(n)\hat{\mathbf{h}}_l \\ &= \Re\{e_{lm}(n)\} + j\Im\{e_{lm}(n)\}, \end{aligned} \tag{3}$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote real and imaginary components respectively. The objective of the complex MCLMS is to adapt simultaneously both the real and the imaginary components of $\hat{\mathbf{h}}$ [8] and consequently, a cost function is formulated as

$$\begin{aligned} J(n) &= \sum_{l=1}^{M-1}\sum_{m=l+1}^{M} e_{lm}(n)e_{lm}^*(n) \\ &= \sum_{l=1}^{M-1}\sum_{m=l+1}^{M} \Re\{e_{lm}(n)\}^2 \\ &\quad + \sum_{l=1}^{M-1}\sum_{m=l+1}^{M} \Im\{e_{lm}(n)\}^2, \end{aligned} \tag{4}$$

where $[\cdot]^*$ denotes complex conjugation.

The optimal estimate of the channels is found by minimising $J(n)$ with respect to $\hat{\mathbf{h}}$,

$$\hat{\mathbf{h}}_{\text{opt}} = \arg\min_{\hat{\mathbf{h}}} E\{J(n)\}, \quad \text{subject to } \|\hat{\mathbf{h}}\| = 1, \tag{5}$$

where $E\{\cdot\}$ is the expectation operator. The unit norm constraint is introduced to avoid the trivial solution $\hat{\mathbf{h}} = \mathbf{0}$. In the case of a channel coefficient vector with complex entries, the constraint only affects the magnitude of the solution and not the phase, which will be elaborated on in Section 4. By enforcing the unit norm constraint at all times, the normalised cost function can be written

$$\tilde{J}(n) = \frac{J(n)}{\|\mathbf{h}\|^2}. \tag{6}$$

The LMS adaptive algorithm finds the desired solution iteratively, where the coefficients are updated according to the relation [1, 8]

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) - \mu\nabla\tilde{J}(n), \tag{7}$$

where ∇ is the gradient operator and μ is a positive step-size. In the case of complex data studied here, the gradient estimate consists of a real and an imaginary component

$$\nabla\tilde{J}(n) = \Re\{\nabla\tilde{J}(n)\} + j\Im\{\nabla\tilde{J}(n)\}. \tag{8}$$

Next, we evaluate each of these components individually. The instantaneous gradient estimate at time n with respect to the real component of the channel vector, $\Re\{\mathbf{h}\}$, is

$$\begin{aligned} \Re\{\nabla\tilde{J}(n)\} &= \frac{\partial}{\partial\Re\{\mathbf{h}\}}\left(\frac{J(n)}{\|\mathbf{h}\|^2}\right) \\ &= \frac{1}{\|\mathbf{h}\|^2}\left[\frac{\partial J(n)}{\partial\Re\{\mathbf{h}\}} - 2\tilde{J}(n)\Re\{\mathbf{h}\}\right] \end{aligned} \tag{9}$$

where

$$\frac{\partial J(n)}{\partial\Re\{\mathbf{h}\}} = \begin{bmatrix} \left(\frac{\partial J(n)}{\partial\Re\{\mathbf{h}_1\}}\right)^T \\ \left(\frac{\partial J(n)}{\partial\Re\{\mathbf{h}_2\}}\right)^T \\ \vdots \\ \left(\frac{\partial J(n)}{\partial\Re\{\mathbf{h}_M\}}\right)^T \end{bmatrix}.$$

Evaluating the partial derivative of $J(n)$ with respect to the real coefficients of the kth channel only results in

$$\frac{\partial J(n)}{\partial\Re\{\mathbf{h}_k\}} = -\sum_{l=1}^{M}\left[\mathbf{x}_l^*(n)e_{ik}(n) + \mathbf{x}_l(n)e_{ik}^*(n)\right]. \tag{10}$$

Similarly, the instantaneous gradient estimate at time n with respect to the imaginary component of the channel vector, $\Im\{\mathbf{h}\}$, is

$$\begin{aligned} \Im\{\nabla\tilde{J}(n)\} &= \frac{\partial}{\partial\Im\{\mathbf{h}\}}\left(\frac{J(n)}{\|\mathbf{h}\|^2}\right) \\ &= \frac{1}{\|\mathbf{h}\|^2}\left[\frac{\partial J(n)}{\partial\Im\{\mathbf{h}\}} - 2\tilde{J}(n)\Im\{\mathbf{h}\}\right] \end{aligned} \tag{11}$$

with

$$\frac{\partial J(n)}{\partial\Im\{\mathbf{h}_k\}} = j\sum_{l=1}^{M}\mathbf{x}_l^*(n)e_{ik}(n) - \mathbf{x}_l(n)e_{ik}^*(n). \tag{12}$$

We then substitute the results from (9), (10), (11) and (12) into (8) in order to obtain the instantaneous estimate of the overall gradient

$$\nabla \tilde{J}(n) = \frac{1}{\|\hat{\mathbf{h}}\|^2}\left[2\mathbf{R}^*(n)\hat{\mathbf{h}}(n) - 2\tilde{J}(n)\mathbf{h}(n)\right], \quad (13)$$

with
$\mathbf{R}(n) =$

$$\begin{bmatrix} \sum_{l\neq 1}\mathbf{R}_{x_l x_l}(n) & -\mathbf{R}_{x_2 x_1}(n) & \cdots & -\mathbf{R}_{x_M x_1}(n) \\ -\mathbf{R}_{x_1 x_2}(n) & \sum_{l\neq 2}\mathbf{R}_{x_l x_l}(n) & \cdots & -\mathbf{R}_{x_M x_2}(n) \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{R}_{x_1 x_M}(n) & -\mathbf{R}_{x_2 x_M}(n) & \cdots & \sum_{l\neq M}\mathbf{R}_{x_l x_l}(n) \end{bmatrix} \quad (14)$$

where

$$\mathbf{R}_{x_l x_m}(n) = \mathbf{x}_l(n)\mathbf{x}_m^H(n) \quad (15)$$

and $\hat{\mathbf{h}}(n) = [\hat{\mathbf{h}}_1^T(n)\ \hat{\mathbf{h}}_2^T(n)\ \ldots\ \hat{\mathbf{h}}_M^T(n)]^T$ is a vector of the concatenated channel estimates at time n.

Finally, substituting (13) into (7) and assuming that the channel estimates are normalised after each iteration as in [1], the update equation for the complex MCLMS algorithm becomes

$$\hat{\mathbf{h}}(n+1) = \frac{\hat{\mathbf{h}}(n) - 2\mu[\mathbf{R}^*(n)\hat{\mathbf{h}}(n) - J(n)\hat{\mathbf{h}}(n)]}{\|\hat{\mathbf{h}}(n) - 2\mu[\mathbf{R}^*(n)\hat{\mathbf{h}}(n) - J(n)\hat{\mathbf{h}}(n)]\|}. \quad (16)$$

The result in (16) differs from the result presented in [1] in that the complex conjugation is applied to the correlation matrix, $\mathbf{R}$. This is also consistent with the result in [8] where complex conjugation is applied to the input data vector.

4. EFFECTS OF THE COMPLEX FACTOR

In this section, the effects of the arbitrary factor on the complex multichannel LMS are formulated and discussed. As will be demonstrated in Section 5, the algorithm from (16) correctly identifies the channels up to a constant factor, c as in the real case. However, now this constant is a complex number, $c = |c|e^{j\theta_c}$ with a magnitude $|c| = \sqrt{\Re\{c\}^2 + \Im\{c\}^2}$ and a phase $\theta_c = \Im\{\ln(c)\}$. Therefore, it introduces an arbitrary additive phase ambiguity in addition to the gain ambiguity occurring for real signals [1, 9]. At convergence the ith coefficient of the lth estimated channel vector is related to the corresponding true channel coefficient according to

$$\hat{h}_{l,i} = |c||h_{l,i}|e^{j\left(\theta_{h_l}(i)+\theta_c\right)}, \quad (17)$$

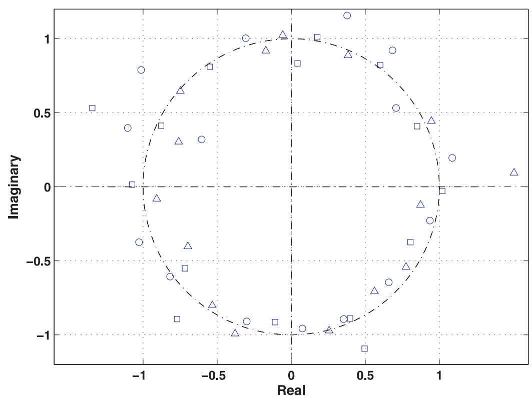

Figure 2: *Channel zeros of a system of M=3 for Channel 1 (squares), Channel 2 (circles) and Channel 3 (triangles).*

where $\theta_{h_l}(i) = \Im\{\ln(h_{l,i})\}$ is the phase of $h_{l,i}$. This makes it difficult to use the estimates in, for example, equalisation where the recovered signal would also have a phase error. Therefore, it is desirable to compensate for the additive angle θ_c. A straightforward approach is to assume knowledge of the phase of one true tap value and compensate for this.

Moreover, we have found through our experiments that if the adaptive algorithm is initialised with a vector whose ith value is set to $\alpha = |\alpha|e^{j\theta_\alpha}$, such that $\hat{\mathbf{h}}(0) = [0\ \ldots\ \alpha\ \ldots\ 0]^T$, the algorithm will converge to a solution where the phase of that same ith value of the estimated channel is equal to θ_α. Consequently, *a priori* knowledge of the phase of one true tap can be used in the initialisation of the algorithm to avoid phase errors. Further work is required to recover the phase of the channels from the observations only.

5. SIMULATIONS

We now present simulation results in order to validate the performance of the proposed algorithm in (16). The Normalised Projection Misalignment (NPM) is chosen as an evaluation metric and is defined as [1]

$$\mathrm{NPM}(n) = 20\log_{10}\left(\frac{\|\mathbf{h} - \beta(n)\hat{\mathbf{h}}(n)\|}{\|\mathbf{h}\|}\right)\mathrm{dB}, \quad (18)$$

with

$$\beta(n) = \frac{\mathbf{h}^T\hat{\mathbf{h}}(n)}{\hat{\mathbf{h}}^T(n)\hat{\mathbf{h}}(n)}. \quad (19)$$

The NPM is a metric which takes into account only the misalignment and not the scaling factor by projecting the true solution vector onto the estimate vector [1]. In the

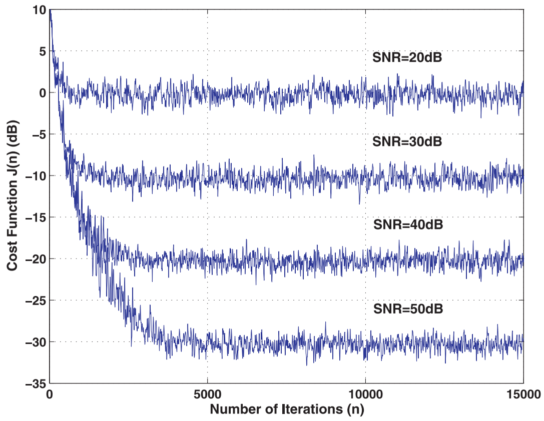

Figure 3: *Cost function trajectory for the Complex MCLMS and for varying levels of SNR with* $M = 3$ *random channels of length* $L = 16$.

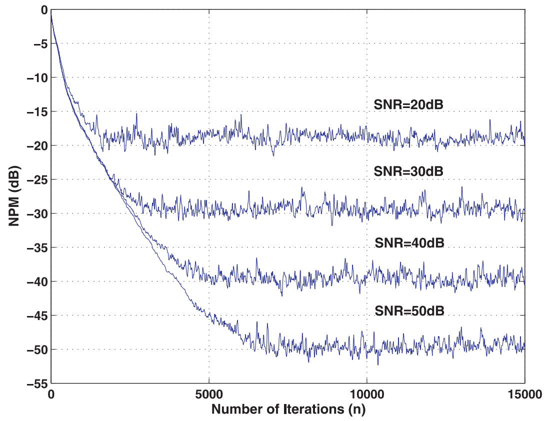

Figure 4: *Normalised projection misalignment for the Complex MCLMS and for varying levels of SNR with* $M = 3$ *random channels of length* $L = 16$.

case of the complex channels considered here, the projection misalignment discounts both the magnitude and the phase of the arbitrary complex factor.

For the experiments we used an example system of $M = 3$ random complex channels of length $L = 16$. The channel zeros are shown in Fig. 2 where squares, circles and triangles indicate each the three different channels. It was assured that there are no common zeros between the channels and the input was complex white Gaussian noise so as to satisfy the identifiability conditions stated in Section 2. The SNR was varied between $20 - 50$ dB and the adaptation step-size was set to $\mu = 10^{-4}$.

The trajectories of the cost function for a typical run and for different noise levels are presented in the plot in Fig. 3. In Fig. 4 the plot shows the corresponding result in terms of NPM. It can be seen that the channels are identified correctly, up to a multiplicative complex factor.

6. CONCLUSIONS

We have derived the complex multichannel LMS algorithm for adaptive blind system identification. This algorithm identifies the unknown system responses up to a multiplicative complex factor, which leads to an arbitrary magnitude and phase ambiguity. The phase ambiguity can be avoided if *a priori* knowledge of the phase of either one of the true channel coefficients is available, however, further work is required to solve this problem blindly. Finally, simulation results verified the performance of the algorithm both in terms of the error trajectory and in terms of normalised projection misalignment.

7. REFERENCES

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